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Credit Scoring – General Approach in the IFRS 9 Context

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Abstract

With the coming into force of the standard IFRS 9 – Financial Instruments, in January 2018, financial institutions passed from an incurred loss model to a forward-looking model for the computation of impairment losses. As such, the IFRS 9 models use point-in-time, estimates of Probability of Default and Loss Given Default and provide a more faithful representation of the credit risk at a given as they are based on past experiences as well as the most recent and forecasted economic conditions. However, given the short-term fluctuations in the macroeconomic conditions, the final outcome of the Expected credit loss models is highly volatile due to their sensitivity to the business cycle. With regard to Probability of Default estimation under IFRS 9, the most commonly methods are: Markov Chains, Survival Analysis and single-factor models (Vasicek and Z-Shift). The development of the score-cards is still the same as in the case of the Internal Ratings Based Probability of Default models, encouraging institutions to use the already available credit rating systems and perform adjustment to the calibration. This paper outlines a non-exhaustive list of quantitative validation tests would satisfy the requirements of the IFRS 9 standard.

Key words*: IFRS 9; credit scoring; statistic tests; financial institutions;*

JEL Classification*: M41, M21*

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1. Introduction

Before the introduction of the IFRS 9 – *Financial Instruments* standard, only sophisticated and complex institutions would have been familiar to rating systems as they would have been used for Internal Ratings Based (IRB) or economic capital models. Hence, before the introduction of IFRS 9, many small and medium sized institutions have simplistic rating methods in place. The existence of an adequate rating system has been discussed by Hamerle et al (2003) who consider that for example a rating system composed of two grades would be considered inappropriate for the computation of the capital requirements. Hence despite IFRS 9 not prescribing the number of grades expected to be included in a rating system it is encouraged to use a similar approach to Basel III, whereas the Bank for International Settlements (BIS) guidelines consider appropriate a minimum of seven performing grades (for retail and non-retail exposures) and at least one default grade. More exactly, credit scoring refers to the set of techniques used by institutions to assess an obligor's creditworthiness by using predictive models to facilitate the credit assessment process the obligor is accepted based on the institution's risk appetite as well as the maximum value that could be lent.

Credit scoring models are used to predict the probability of obligors' default. To measure the quality of the scoring models quantitative tests can be use such as: Gini index, AUROC, Somers' D, KS statistics, Information statistics (Information Value, Weight of Evidence), Binomial test, Chi-square test, Population Stability Index, Herfindahl-Hirschman Index (HHI). Such techniques can be used throughout the selection process, as well as for validation and monitoring purposes to assess the quality of the mode after its deployment.

The paper will present a non-exhaustive list of statistic tests used in credit scoring validation including a short description and advantages and disadvantages. Institutions can apply a large variety of methods when validating their rating models, however it is essential that at least one method is applied for each validation layer discrimination, calibration, stability and concentration.

2. Literature review

Credit scoring is considered one of the most essential methods used by banks, following the rapid expansion of the credit industry worldwide. It is heavily used by financial institutions to provide credit to good applicants and to differentiate good credit from bad credit. The decision that involves granting or refusing to grant credit to a client can also be supported by court techniques, which, according to Sarlija et al. (2004), are based on previous or current knowledge and experiences of credit analysts, the latter evaluate clients in terms of credit repayment capacity, guarantees. Although these judgment techniques can also be applied, financial institutions use, rather, credit rating models, out of the need to quantify credit risk. Gup and Kolari (2005) define credit scoring as a use of statistical models in order to establish the probability that a potential borrower will no longer be granted a loan. According to the same authors, rating models are used to evaluate business loans, real estate and consumption. Thomas et al (2002) consider credit scoring as a set of decision models that help creditors in granting consumer loans: who will receive credit, what operational strategies can increase the profitability of lenders to creditors. Crook (1996) presents a number of benefits of credit scoring. One of the most important advantages is that, in order to make a decision, a smaller volume of information is needed, because credit scoring models have been estimated to include only variables that are correlated with repayment performance. At the same time, through credit scoring attempts are made to correct any prejudices that may result from taking into account the reimbursement history only for the approved applications.

3. Research methodology

The research methodology aims a deductive approach which highlights the theoretical perspective regarding the concept of IFRS 9 and a non-exhaustive list of statistic tests used in credit scoring validation. As research method can have mentioned the documents analyze which consists in going through the specialized literature in order to identify the relevant works to the examined subject. Were accessed books and articles from the field, European regulations and International Accounting Standards (International Financial Reporting Standard 9 - *Financial Instruments*) and, also, web pages of the Basel Committee on Banking Supervision and European Banking Authority.

4. Statistic tests used in credit scoring validation

4.1. Kolmogorov-Smirnov (KS)

Among the most commonly used test for the assessment of the discriminatory power of a model is the Kolmogorov-Smirnov statistic by quantifying the distance between two distributions good and bad observations as per the definitions established by the institution. The KS values can range between 0 and 1, where 1 implies that the model is able to accurately distinguish between the good and bad populations. Hence, the higher the KS the better the model.

For each individual obligor, where a score S is available, the following is applicable:

Equation 1

$$D_K = \begin{cases} 1, client\ is\ good \\ 0, \quad otherwise \end{cases}$$

Using the aforementioned formula, the empirical cumulative distribution functions (CDF) of scores of good, bad or all can be computed:

Equation 2

$$F_{GOOD}(a) = \frac{1}{n}\sum_{i=1}^{n} I(s_i \leq a | D_K = 1), \qquad a \in [L,H]$$

$$F_{BAD}(a) = \frac{1}{m}\sum_{i=1}^{m} I(s_i \leq a | D_K = 0), \quad a \in [L,H]$$

$$F_{ALL}(a) = \frac{1}{n+m}\sum_{i=1}^{n+m} I(s_i \leq a), \qquad a \in [L,H]$$

Where:

s_i – Score of the i^{th} obligor;

n – Number of good obligors;

m – Number of bad obligors;

I – indicator function where I (true) = 1 and I (false) = 0;

L – The minimum value of given score;

H – The maximum value of given score.

Based on Equation 2, the KS is defined as follows:

Equation 3

$$KS = max_{a \in [L,H]} |F_{BAD}(a) - F_{GOOD}(a)|$$

The KS statistic should be computed both for the development and validation samples. Furthermore, it is expected that the institution sets in place a monitoring framework to enable a timely detection of any degradations of the discriminatory power by computing the KS for each subsequent sample the quarterly values can be assessed against the initial validation value. Depending on the nature, size and specific characteristics of the portfolio as well as considering the institution's risk appetite and regulatory constraints, the institution should define the thresholds for the KS tests. Most often they are also associated with the traffic lights presentation.

4.2. Lorenz Curve (LC) and Gini

The Lorenz curve (LC), Cumulative Accuracy Profile (CAP) or Accuracy Ratio (AR) is a statistical test used to assess the discriminatory power of the risk ranking mechanism (scoring function) as it reflects the ability to dissociate between good and bad obligors (the relationship between cumulative distribution function for good and bad obligors).

The first step of the process is to order all obligors based on the scores predicted by the model, from the lowest probability to the highest. The percentage of defaulted borrowers within each probability band is projected from the lower probability to the maximum probability. The Gini is defined as the ratio of the area between the cumulative function of the model and the cumulative function of the random model and the area between the cumulative function of the perfect model. This ideal model will give the perfect discrimination between pools, assigning events in the desired proportion according to the pools ranking a higher probability of default for ratings as they are closer to default rating.

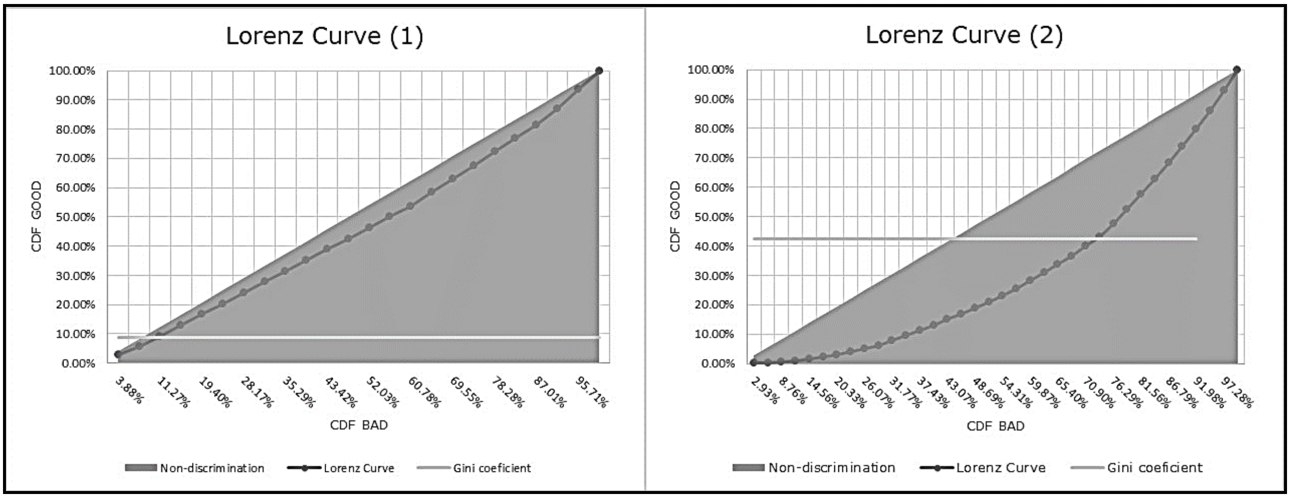
As in the case of the KS, the parametrical function is defined as follows:

x = $F_{BAD}(a)$, $a \in [L,H]$

y = $F_{GOOD}(a)$, $a \in [L,H]$

Figure no. 1 presents the results of two models: model A, which had a low discriminatory power, as it is closer to the hypotenuse, and model B, which has a high discriminatory power. In case of a random model (no discriminatory power), the plotted curve would cut the graph in half making a 45-degree angle with the Cartesian axes, while a perfect model would present a curve that will exactly delimit the two adjacent of the triangle.

Figure no. 1. Lorenz curve



Source: Own processing, 2021

The Gini index describes the overall quality of the risk ranking mechanism scoring function which takes values between -1 and 1 under the ideal model the scoring function separates between good and bad obligors, hence the Gini is close to 1while a random model assigning a random score would have n Gini close to 0. In the case of negative Gini values, the values correspond to a model with reversed meaning of scores. Please refer to **Appendix 1** for the underlying data and computation.

The Gini is computed taking into account the bad and good continuous cumulative distributions (F_{BAD}) (F_{GOOD}):

Equation 4

$$Gini = 2\int_0^1 F_{BAD}\, dF_{GOOD} - 1$$

For a discrete approximation the Trapezoidal rule or Simpson's rule is used:

$$Gini \approx \sum_{k=1}^{N}\left[\left(F_{BAD_k} + F_{BAD_{k-1}}\right) * \left(F_{GOOD_k} - F_{GOOD_{k-1}}\right)\right] - 1$$

Where N is the number of observations of BAD and GOOD distributions analysed.

As previously mentioned, the tests should be performed on the development and validation samples. A degradation in their outputs is expected, especially for the out-of-sample and out-of-time samples, however the differences should be within the accepted tolerance level. Hence, institutions should set a monitoring framework to identify early stages of deterioration. It is recommended that the test is performed for both monitoring purposes and annual validation. The thresholds for the test must be set in accordance with the specificity of the portfolio; lower values are expected for corporate and SME portfolio than for residential real estate portfolios.

4.3. Receiver Operating Characteristic curve (ROC)

The ROC depicts the relation between the complements of two CDFs. Given two continuous random variables, X and Y, a point on a graph of an ROC curve is located at P(X ≥ ρ) on the horizontal axis and at P(Y ≥ ρ) on the vertical axis, where "ρ" is a constant that can take on values within the domain of X and Y. In relation to the ROC curve the following concepts are defined:

- True positive rate – hit rate (HR) represents the sensitivity or the events correctly identified;
- True negative rate – specificity represent the non-events correctly identified;
- False negative rate –1-specificity represents false alarm rate (FAR);
- False positive rate –1-sensitivity.

The ROC curve is obtained by plotting HR against FAR for different values of "ρ". Hence ROC curve close to the diagonal (non-discrimination line), indicates a random model, while a curve close to the top left corner presents a model with a high discriminatory power, hence the greater the area under the ROC curve, the better the model.

This leads to another test for assessing the discriminatory power of the model: area under curve (AUC or AUROC) also called coefficient of concordance (c). A value of 0.5 depicts a random model while a value of 1 indicates that ROC curve lies in the top left corner and model is discriminating perfectly.

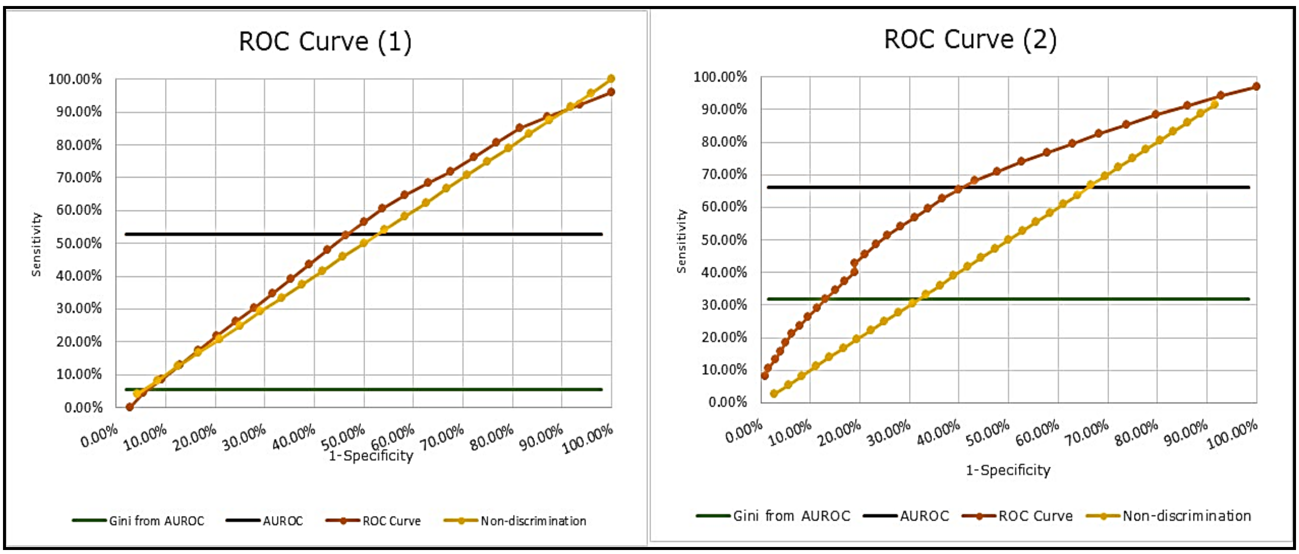
The Gini can be expressed as AUROC:

Equation 5

$$Gini = 2 * AUROC - 1$$

The graphs in ***Figure no. 2*** describes two ROC curves: the one on the right reveals a higher discrimination power for its underlying model than the one on the left.

Figure no. 2. ROC curves



Source: Own processing, 2021

For an exact result, integral calculus is recommended; however, the Trapezoidal or Simpson's rule can be used to approximate AUROC:

$$AUROC_{theoretical} = 1 - \frac{1}{\widehat{D}(N - \widehat{D})} \sum_{i=1}^{K} (N_i - \widehat{D}_i)\left(\frac{\widehat{D}_i}{2} + \sum_{j=1}^{i-1} \widehat{D}_j\right)$$

Where:

N – Number of customers in the portfolio at the beginning of the observation period;

K – Rating grades for non-defaulted exposures;

N_i – The number of customers in the i-th rating grade;

PD_i – PD used to estimate future defaults for the i-th rating grade;

$\widehat{D}_i = [N_i * PD_i]$ – Estimated number of default for rating grade I, where [.] denotes the nearest integer;

$\widehat{D} = \sum_{i=1}^{K} \widehat{D}_i$ – The estimated total number of defaults.

As it can be seen from the above presented example the model used to generate Curve 2 has a better performance than the one used to generate Curve 1.

4.4. Somers' D (D_s)

Somers' D is an ordinal measure which can be defined in terms of Kendall'sτ_a. It must be mentioned that the Gini index is a special case of Somers' D.

Given a sequence of bivariate random variables (X,Y) = {(X_i, Y_i)} Kendall's τ_a is defined as:

$$\tau(X,Y) = E\left[sign(X_i - X_j)sign(Y_i - Y_j)\right]$$

Where:

E[.] – denotes expectation;

(X_i, Y_i) and (X_j, Y_j): represent the bivariate random variables extracted independently from the same underlying population for PD models X = 1 good and X = 0 if bad, Y represents scores.

Kendall's τ_a shows us the difference between the probability that the two (X,Y) pairs are concordant and the probability that the two (X,Y) pairs are discordant.

Somers' D of a given credit scoring model, denoted as D_s is calculated as follows:

$$D_s = \frac{\sum_i g_i \sum_{j<i} b_j - \sum_i g_i \sum_{j>i} b_j}{n * m}$$

Where:

g_i (b_j) – is number of goods (bads) in i^{th} interval of scores;

n – Number of good;

m – Number of bad.

In other words, less mathematically, Somers' D is defined as:

$$D_s = \frac{ConcordantPairs - DiscordantPairs}{TotalNumberofPairsIncludingTies}$$

Another way to calculate D_s is by Mann-Whitney U-statistic. In order to compute this statistic, the sample must be order in an increasing manner by score value; sum ranks of goods must be performed, let this beR_G.

The D_s is given by:

$$D_s = 2 * \frac{U}{n * m} - 1$$

Where U is given by $U = R_G - \frac{1}{2} * n * (n+1)$

Mathematically Somers' D is equal to Gini index. However, due to that fact that Somers' D is more resource intensive and complex (portfolio size defines the number of potential good/bad pairs) and cannot be properly approximated, Gini is more commonly used. Somers' D analysis is carried out using a software provided by SAS which is commonly used across many financial institutions.

4.5. Information value (IV)

When building a scorecard, the Information Value (IV) statistic is a popular method for selecting predictor variables. Given that default status can be modeled as a binary outcome IV is a good way to assess the predictor power.

Taking into account that the target is binary, when constructing explanatory variables for a scorecard development, continuous variables cannot be easily validated. Thus, each predictor X observations must be grouped. The number of grades should ensure the correlation between the explanatory and the target variable is relevant. Upon the removal of outliers and identification of trends IV decreases with the decreases of grades.

4.6. Herfindhal Hirschman Index (HHI)

Credit portfolio models must ensure the homogeneity of exposures within the same grade and heterogeneity between grades. The Herfindhal-Hirschman Index (HHI) is amongst the most common tests used to ensure that the portfolio's segmentation is adequate; none of the grades present high concentration:

$$HHI = \sum_{i=1}^{n} \gamma_i^2$$

Where:

n – Number of facilities/exposures in the portfolio;

γ_i - The exposure of facility "I" relative to the portfolio's total value.

The above formula is applied at portfolio level. However, in order to identify the high/low concentration within each grade the following formula must be applied: HHI to increase, it is useful to have a segmented view. It can be mathematically/expert based assumed that the portfolio is divided in "m" buckets.

$$HHI = \sum_{k=1}^{m} F_k^2 H_k$$

Where:

M- Number of grades;

F_k- Total exposure amount of grade "k" relative to the total value of the portfolio;

H_k-The HHI in each bucket.

HHI can be used to assess concentration in the distribution of obligors/facilities in grades or pools. Firstly, a coefficient of variation is calculated, then the Herfindahl index:

$$CV_{curr} = \sqrt{K \sum_{i=1}^{K} \left(R_i - \frac{1}{K}\right)^2}$$

$$HHI = \frac{CV_{curr}^2 + 1}{K}$$

Where:

K – Number of rating grades for non-defaulted exposures;

R_i – Relative frequency of rating grade "i" at the beginning of the relevant observation period.

4.7. Bootstrap validation

In case the institution has a low sample or short observation period Bootstrapping is used in order to simulate the out-of-sample characteristic of the population. Bootstrap can act as a sampling method and a prerequisite of the validation framework in accordance with the law of large numbers and cannot be considered a standalone method of validation. After the sampling, given sufficient data and concordance between validation test and estimation method, all other statistical tests can be computed (Gini, KS, AUROC, Binomial etc.). Suffice to say, bootstrap validation can be used in a multiple manner. The main drawback of bootstrapping is that it often is computationally expensive.

5. Conclusions

This paper outlines the key quantitative techniques institutions should use to assess the adequacy of its credit-scoring framework for assigning credit grades to exposures and also the benefits of credit scoring. It outlines tests that institutions should use to assess the predictively of its models and how representative the population of exposures the model was developed on is to the population of exposures that the model is applied to. However, it is not simply sufficient for the institutions to perform these tests, they must also ensure that they have suitable thresholds in place to identify when a model fails these tests and ensure appropriate action is undertaken to remediate when a model fails the test. Where possible the tests outlined in this section should be applied at factor level as well at overall grade level to allow the institution to identify any deterioration in performance in an individual factor that may not be evident when tests are performed at an overall credit grade level. Finally, these tests are of upmost importance in determining the adequacy of the rank ordering of the credit grades, however they do not assess the adequacy of the quantification of the provision estimates. These should be assessed at an overall PD parameter level after the model has been calibrated to 1 year PIT PD's for stage 1 estimates and lifetime PD's for stage 2 estimates.

The benefits of credit scoring are as follows: it is requiring less information to make a decision, because the models have been estimated to take in consideration only those variables, which are statistically correlated with the repayment performance. Credit scoring models take in consideration the aspects of good as well as bad payers. Credit scoring models are based on larger time and information samples than an analyst can remember.

REFERENCES

1. Basel committee on banking supervision (2015), Guidance on credit risk and accounting for expected credit losses, https://www.bis.org/bcbs/publ/d350.pdf
2. Borio C., Lowe P. (2001), To provision or not to provision, *BIS Quarterly Review*, Vol. 9, No. 3, pp. 36-48.
3. Bushman R.M., Williams C.D. (2015), Delayed expected loss recognition and the risk profile of banks, *Journal of Accounting Research*, Vol. 53, No. 3, pp. 511-553.
4. Carlehed M., Petrov A. (2012), A methodology for point-in-time-through-the-cycle probability of default decomposition in risk classification systems, *Journal of Risk Model Validation*, Vol. 6, No. 3, pp. 3-25.
5. Chawla, G., Forest L.R., Aguais S.D. (2016), Point-in-time loss-given default rates and exposures at default models for IFRS 9/CECL and stress testing, *Journal of Risk Management in Financial Institutions*, Vol. 9, No. 3, pp. 249-263.
6. Crook, J. N. (1996), Credit scoring: An overview. *Working paper series No. 96/13, British*
7. *Association, Festival of Science. University of Birmingham, The University of Edinburgh.*
8. European Banking Authority (2017), Guidelines on credit institutions' credit risk management practices and accounting for expected credit losses, https://www.eba.europa.eu/sites/default/documents/files/documents/10180/1842525/d769d006-d992-4202-8838-711a034e80a2/Final%20Guidelines%20on%20Accounting%20for%20Expected%20Credit%20Losses%20%28EBA-GL-2017-06%29.pdf
9. European Banking Authority (2017), Guidelines on PD estimation, LGD estimation and the treatment of defaulted exposures, https://www.eba.europa.eu/sites/default/documents/files/documents/10180/2033363/6b062012-45d6-4655-af04-801d26493ed0/Guidelines%20on%20PD%20and%20LGD%20estimation%20%28EBA-GL-2017-16%29.pdf?retry=1
10. European Banking Authority (2016), Guidelines on the application of the definition of default under Article 178 of Regulation (EU) No. 575/2013, https://www.eba.europa.eu/sites/default/documents/files/documents/10180/1597103/004d3356-a9dc-49d1-aab1-3591f4d42cbb/Final%20Report%20on%20Guidelines%20on%20default%20definition%20%28EBA-GL-2016-07%29.pdf?retry=1
11. Global Public Policy Committee (2016), The implementation of IFRS 9 impairment requirements by banks, https://www.iasplus.com/en/publications/global/other/ifrs-9-impairment-banks
12. Gordy, M. (2003), A Risk-Factor Model Foundation for Ratings-Based Bank Capital Rules, *Journal of Financial Intermediation*, Vol. 12, pp. 199-232.
13. Gordy, M. (2000), A Comparative Anatomy of Credit Risk Models, *Journal of Banking and Finance*, Vol. 24, pp. 119-149.
14. Gup, B. E., Kolari, J. W. (2005), Commercial Banking: The management of risk. *Alabama: John Wiley&Sons, Inc.*
15. Hamerle, A., Liebig T., Rosch D. (2003), Benchmarking Asset Correlations Risk, Vol. 16, pp. 77-81.
16. Heitfield, A. (2005), Dynamics of Rating Systems, in: Basel Committee on Banking Supervision: Studies on the Validation of Internal Rating Systems, *Working Paper No. 14*, February, pp. 10-27.
17. Gebhardt, G.U., Novotny-Farkas, Z. (2011), Mandatory IFRS adoption and accounting quality of European banks, *Journal of Business Finance&Accounting*, Vol. 38, No.3-4, pp. 289-333.
18. International accounting standards Board (2014), International Financial reporting standard 9 Financial instruments, https://www.ifrs.org/issued-standards/list-of-standards/ifrs-9-financial-instruments/
19. Novotny-Farkas, Z. (2016), The interaction of the IFRS 9 expected loss approach with supervisory

rules and implications for financial stability, *Accounting in Europe*, Vol. 13, No. 2, pp. 197-227.

20. Onali, E., Ginesti, G. (2014), Pre-adoption market reaction to IFRS 9: a cross-country event-study, *Journal of Accounting and Public Policy*, Vol. 33, No. 6, pp. 628-637.
21. Peduzzi, P., Concato, J., Feinstein, A.R., Holford, T.R. (1995), Importance of events per independent variable in proportional hazards regression analysis II. Accuracy and precision of regression estimates, *Journal of Clinical Epidemiology*, Vol. 48, No. 12, pp. 1503-1510.
22. Pool, S., De Haan, L., Jacobs, J.P. (2015), Loan loss provisioning, bank credit and the real economy, *Journal of Macroeconomics*, Vol. 45, No. 1, pp. 124-136.
23. Reitgruber, W. (2016), Expected loss provisioning under upcoming IFRS 9 impairment standards: a new source of P&L volatility – Can we tame it?, *Journal of Risk Management in Financial Institutions*, Vol. 9, No. 4, pp. 332-343.
24. Sarlija, N., Bensic M., Bohacek Z. (2004), Multinomial Model in Consumer Credit Scoring, *10th International Conference on Operational Research*.
25. Skoglund, J. (2017), Credit risk term-structures for life time impairment forecasting: a practical guide, *Journal of Risk Management in Financial Institutions*, Vol. 10, No. 2, pp. 177-195.
26. Thomas, L. C., Edelman, D. B., Crook, L. N. (2002), Credit Scoring and Its Applications.
27. *Philadelphia: Society for Industrial and Applied Mathematics*.
28. Yang, B.H. (2017), Point-in-time PD term structure models for multi-period scenario loss projection: methodologies and implementations for IFRS 9 ECL and CCAR stress testing, *Journal of Risk Model Validation*, Vol. 11, No. 3, pp. 1-17.

Appendix 1 – Lorenz Curve and ROC data

Generation of curve 1 - The data used is randomly generated in order to illustrate a ranking system composed of 24 grades.

	Ascending BAD							
Rating	GRADE	GOOD	BAD	PDF_BAD	CDF_BAD	PDF_GOOD	CDF_GOOD	
A	1	162	3	2.78%	2.78%	3.88%	3.88%	
	2	157	3	2.78%	5.56%	3.76%	7.65%	
	3	151	4	3.70%	9.26%	3.62%	11.27%	
	4	155	4	3.70%	12.96%	3.72%	14.98%	
	5	184	4	3.70%	16.67%	4.41%	19.40%	
	6	184	4	3.70%	20.37%	4.41%	23.81%	
B	7	182	4	3.70%	24.07%	4.36%	28.17%	
	8	147	4	3.70%	27.78%	3.52%	31.70%	
	9	150	4	3.70%	31.48%	3.60%	35.29%	
	10	168	4	3.70%	35.19%	4.03%	39.32%	
	11	171	4	3.70%	38.89%	4.10%	43.42%	
	12	178	4	3.70%	42.59%	4.27%	47.69%	
C	13	181	4	3.70%	46.30%	4.34%	52.03%	
	14	182	4	3.70%	50.00%	4.36%	56.39%	
	15	183	4	3.70%	53.70%	4.39%	60.78%	
	16	182	5	4.63%	58.33%	4.36%	65.14%	
	17	184	5	4.63%	62.96%	4.41%	69.55%	
	18	180	5	4.63%	67.59%	4.32%	73.87%	
D	19	184	5	4.63%	72.22%	4.41%	78.28%	
	20	184	5	4.63%	76.85%	4.41%	82.69%	
	21	180	5	4.63%	81.48%	4.32%	87.01%	
	22	182	6	5.56%	87.04%	4.36%	91.37%	
	23	181	7	6.48%	93.52%	4.34%	95.71%	
	24	179	7	6.48%	100.00%	4.29%	100.00%	

GRADE	CDF_GOOD	CDF_BAD	Lorenz Curve	Gini coeficient	KS statistic
1	3.88%	2.78%	0.00%	8.73%	7.07%
2	7.65%	5.56%	0.16%	8.73%	7.07%
3	11.27%	9.26%	0.43%	8.73%	7.07%
4	14.98%	12.96%	0.84%	8.73%	7.07%
5	19.40%	16.67%	1.49%	8.73%	7.07%
6	23.81%	20.37%	2.31%	8.73%	7.07%
7	28.17%	24.07%	3.28%	8.73%	7.07%
8	31.70%	27.78%	4.19%	8.73%	7.07%
9	35.29%	31.48%	5.26%	8.73%	7.07%
10	39.32%	35.19%	6.60%	8.73%	7.07%
11	43.42%	38.89%	8.12%	8.73%	7.07%
12	47.69%	42.59%	9.86%	8.73%	7.07%
13	52.03%	46.30%	11.79%	8.73%	7.07%
14	56.39%	50.00%	13.89%	8.73%	7.07%
15	60.78%	53.70%	16.16%	8.73%	7.07%
16	65.14%	58.33%	18.61%	8.73%	7.07%
17	69.55%	62.96%	21.28%	8.73%	7.07%
18	73.87%	67.59%	24.10%	8.73%	7.07%
19	78.28%	72.22%	27.18%	8.73%	7.07%
20	82.69%	76.85%	30.47%	8.73%	7.07%
21	87.01%	81.48%	33.89%	8.73%	7.07%
22	91.37%	87.04%	37.56%	8.73%	7.07%
23	95.71%	93.52%	41.48%	8.73%	7.07%
24	100.00%	100.00%	45.63%	8.73%	7.07%

For the generation of Curve 2 a random simulated portfolio was used, the portfolio was grouped into 36 grades, as follows:

	Ascending BAD						
Rating	GRADE	GOOD	BAD	PDF_BAD	CDF_BAD	PDF_GOOD	CDF_GOOD
A	1	422	4	0.05%	0.05%	2.93%	2.93%
	2	421	12	0.15%	0.20%	2.92%	5.85%
	3	419	24	0.30%	0.50%	2.91%	8.76%
	4	418	36	0.45%	0.94%	2.90%	11.67%
	5	417	44	0.55%	1.49%	2.90%	14.56%
	6	416	56	0.69%	2.18%	2.89%	17.45%
B	7	415	64	0.79%	2.98%	2.88%	20.33%
	8	414	76	0.94%	3.92%	2.87%	23.21%
	9	413	84	1.04%	4.96%	2.87%	26.07%
	10	411	104	1.29%	6.25%	2.85%	28.93%
	11	409	124	1.54%	7.79%	2.84%	31.77%
	12	408	136	1.69%	9.47%	2.83%	34.60%
C	13	407	144	1.79%	11.26%	2.83%	37.43%
	14	407	144	1.79%	13.05%	2.83%	40.25%
	15	405	156	1.93%	14.98%	2.81%	43.07%
	16	405	160	1.98%	16.96%	2.81%	45.88%
	17	405	164	2.03%	19.00%	2.81%	48.69%
	18	405	164	2.03%	21.03%	2.81%	51.50%
D	19	404	172	2.13%	23.16%	2.81%	54.31%
	20	401	196	2.43%	25.60%	2.78%	57.09%
	21	400	212	2.63%	28.22%	2.78%	59.87%
	22	398	224	2.78%	31.00%	2.76%	62.63%
	23	398	224	2.78%	33.78%	2.76%	65.40%
	24	398	224	2.78%	36.56%	2.76%	68.16%
E	25	394	264	3.27%	39.83%	2.74%	70.90%
	26	393	276	3.42%	43.25%	2.73%	73.63%
	27	384	360	4.46%	47.72%	2.67%	76.29%
	28	380	396	4.91%	52.63%	2.64%	78.93%
	29	378	416	5.16%	57.79%	2.62%	81.56%
	30	378	416	5.16%	62.95%	2.62%	84.18%
F	31	376	436	5.41%	68.35%	2.61%	86.79%
	32	376	436	5.41%	73.76%	2.61%	89.40%
	33	371	484	6.00%	79.76%	2.58%	91.98%
G	34	378	514	6.37%	86.14%	2.62%	94.60%
	35	385	544	6.75%	92.88%	2.67%	97.28%
	36	392	574	7.12%	100.00%	2.72%	100.00%

GRADE	CDF_GOOD	CDF_BAD	Lorenz Curve	Gini coeficient	KS statistic
1	2.93%	0.05%	0.00%	42.40%	31.65%
2	5.85%	0.20%	0.00%	42.40%	31.65%
3	8.76%	0.50%	0.01%	42.40%	31.65%
4	11.67%	0.94%	0.03%	42.40%	31.65%
5	14.56%	1.49%	0.07%	42.40%	31.65%
6	17.45%	2.18%	0.12%	42.40%	31.65%
7	20.33%	2.98%	0.20%	42.40%	31.65%
8	23.21%	3.92%	0.30%	42.40%	31.65%
9	26.07%	4.96%	0.42%	42.40%	31.65%
10	28.93%	6.25%	0.58%	42.40%	31.65%
11	31.77%	7.79%	0.78%	42.40%	31.65%
12	34.60%	9.47%	1.03%	42.40%	31.65%
13	37.43%	11.26%	1.32%	42.40%	31.65%
14	40.25%	13.05%	1.66%	42.40%	31.65%
15	43.07%	14.98%	2.06%	42.40%	31.65%
16	45.88%	16.96%	2.51%	42.40%	31.65%
17	48.69%	19.00%	3.01%	42.40%	31.65%
18	51.50%	21.03%	3.58%	42.40%	31.65%
19	54.31%	23.16%	4.20%	42.40%	31.65%
20	57.09%	25.60%	4.87%	42.40%	31.65%
21	59.87%	28.22%	5.62%	42.40%	31.65%
22	62.63%	31.00%	6.44%	42.40%	31.65%
23	65.40%	33.78%	7.34%	42.40%	31.65%
24	68.16%	36.56%	8.31%	42.40%	31.65%
25	70.90%	39.83%	9.35%	42.40%	31.65%
26	73.63%	43.25%	10.49%	42.40%	31.65%
27	76.29%	47.72%	11.70%	42.40%	31.65%
28	78.93%	52.63%	13.02%	42.40%	31.65%
29	81.56%	57.79%	14.47%	42.40%	31.65%
30	84.18%	62.95%	16.06%	42.40%	31.65%
31	86.79%	68.35%	17.77%	42.40%	31.65%
32	89.40%	73.76%	19.63%	42.40%	31.65%
33	91.98%	79.76%	21.60%	42.40%	31.65%
34	94.60%	86.14%	23.78%	42.40%	31.65%
35	97.28%	92.88%	26.17%	42.40%	31.65%
36	100.00%	100.00%	28.80%	42.40%	31.65%